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Numerical Methods - MA 207
Numerical Solutions of Ordinary Differential Equations

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1. Compute $y(0.1)$ and $y(0.2)$ by Runge-Kutta method of fourth order for the differential equation
$$\frac{dy}{dx} = xy + y^2, \quad y(0) = 1.$$
2. Use Runge-Kutta method of fourth order to find $y(0.1)$, given that
$$\frac{dy}{dx} = \frac{1}{x+y}, \quad y(0) = 1.$$
3. Given $y' = x^2 - y,\ y(0) = 1$, find $y(0.1)$ using Runge-Kutta method of fourth order.
4. Using 4th order Runge-Kutta method, find $y(0.1), y(0.2)$ and $y(0.3)$, given that
$$\frac{dy}{dx} = 1 + xy, \quad y(0) = 2.$$
5. Using 4th order Runge-Kutta method, evaluate the value of y when $x = 1.1$, given that
$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}, \quad y(1) = 1.$$
6. Apply third order Runge-Kutta method to find an approximate value of y when $x = 0.2$, given that
$$\frac{dy}{dx} = x + y, \quad y(0) = 1.$$
7. Using Runge-Kutta method of fourth order, solve
$$\frac{dy}{dx} = \frac{y^2 - x^2}{y^2 + x^2}$$
with $y(0) = 1$ at $x = 0.2,\ 0.4$.
8. Apply Runge-Kutta method to find an approximate value of y when $x = 0.2$ in steps of 0.1 if
$$\frac{dy}{dx} = x + y^2$$
given that $y = 1$ where $x = 0$.
9. Using Runge-Kutta method of fourth order, solve for y at $x = 1.2, 1.4$ from
$$\frac{dy}{dx} = \frac{2xy + e^x}{x^2 + xe^x}$$
given $x_0 = 1, y_0 = 0$.

10. Given
$$\frac{dy}{dx} = y - x$$
where $y(0) = 2$, find $y(0.1)$ and $y(0.2)$ correct to 4 decimal places by
   - Runge-Kutta second order formula
   - Runge-Kutta fourth order formula.

11. Given
$$\frac{dy}{dx} = 1 + y^2$$
where $y(0) = 0$, find $y(0.2), y(0.4)$ and $y(0.6)$ by Runge-Kutta fourth order formula.

12. Taking $h = 0.5$, solve the initial value problem for $x = 0.05$ of the differential equation
$$\frac{dy}{dx} = 3x + \frac{y}{2}, \quad y(0) = 1.$$
   - Euler's method
   - Modified Euler's method
   - Runge-Kutta method of order 4.

13. Solve the system of differential equations
$$\frac{dy}{dx} = xz + 1, \qquad \frac{dz}{dx} = -xy$$
for $x = 0.3(0.3)0.9$ using Runge-Kutta fourth order formula. Initial values are $x = 0, y = 0, z = 1$.

14. Using Runge-Kutta method of order 4, find the approximate values of x and y at $t = 0.2$ for the following system:
$$\frac{dx}{dt} = 2x + y, \qquad \frac{dy}{dt} = x - 3y$$
given that when $t = 0$, $x = 0, y = 0.5$.

15. Given
$$\frac{d^2y}{dx^2} - y^3 = 0, \quad y(0) = 10, \quad y'(0) = 5.$$
Evaluate $y(0.1)$ using Runge-Kutta method.

16. Use the Runge-Kutta method with fourth order accuracy to determine the approximate value of y at $x = 0.1$ if y satisfies the differential equation
$$\frac{d^2y}{dx^2} - x^2\frac{dy}{dx} - 2xy = 1, \quad y(0) = 1, \quad y'(0) = 0.$$

17. Using Runge-Kutta method, solve
$$\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$$
for $x = 0.2$ correct to 4 decimal places. Initial conditions are $y(0) = 1, y'(0) = 0$.

18. Given
$$\frac{dy}{dx} = \frac{1}{x+y}, \quad y(0) = 2.$$
If $y(0.2) = 2.09, y(0.4) = 2.17$ and $y(0.6) = 2.24$, find $y(0.8)$ using Milne's method.

19. Using Milne's predictor-corrector formula, find $y(0.4)$, for the differential equation
$$\frac{dy}{dx} = 1 + xy, \quad y(0) = 2.$$

20. Given
$$\frac{dy}{dx} = \frac{1}{2}(1+x^2)y^2$$
and $y(0) = 1, y(0.1) = 1.6, y(0.2) = 1.12, y(0.3) = 1.21$. Evaluate $y(0.4)$ by Milne's predictor-corrector method.

21. Find by Milne's method for the equation
$$y' = y - x^2, \quad y(0) = 1,$$
by obtaining the starting values by Taylor's series method.

22. Using Adams-Bashforth method find $y(4.4)$ given
$$5xy' + y^2 = 2, \quad y(4) = 1, \quad y(4.1) = 1.0049, \quad y(4.2) = 1.0097$$
and $y(4.3) = 1.0143$.

23. Using Adams-Bashforth method determine $y(1.4)$ given that
$$y' - x^2y = x^2, \quad y(1) = 1.$$
Obtain the starting values from the Euler's method.

24. Using Adams-Bashforth method find $y(0.4)$ given that
$$y' = 1 + xy, \quad y(0) = 2.$$

25. Apply Milne's method to find $y(0.4)$ of the initial value problem
$$y' = x - y^2, \quad y(0) = 1.$$
Starting solutions required are to be obtained using Runge-Kutta method of order 4 using step value $h = 0.1$.

26. Using Milne's method, find $y(4.5)$ given
$$5xy' + y^2 - 2 = 0$$
with $y(4) = 1, y(4.1) = 1.0049, y(4.2) = 1.0097, y(4.3) = 1.0143, y(4.4) = 1.0187$.

27. Given
$$y' = x(x^2 + y^2)e^{-x}, \quad y(0) = 1,$$
find y at $x = 0.1, 0.2,$ and 0.3 by Taylor's series method and compute $y(0.4)$ by Milne's method.

28. Using Runge-Kutta method of order 4, find y for $x = 0.1, 0.2, 0.3$ given that

$$\frac{dy}{dx} = xy + y^2, \quad y(0) = 1.$$

Continue the solution at $x = 0.4$ using Milne's method.

29. If

$$\frac{dy}{dx} = 2e^x y, \quad y(0) = 2,$$

find $y(4)$ using Adams predictor-corrector formula by calculating $y(1), y(2)$ and $y(3)$ using Euler's modified formula.

30. Given

$$y'' + xy' + y = 0, \quad y(0) = 1, \quad y'(0) = 0$$

obtain y for $x = 0.1, 0.3$ by any method. Further, continue the solution by Milne's method to calculate $y(0.4)$.

31. Given

$$\frac{dy}{dx} = 1 + y^2$$

where $y = 0$ when $x = 0$. Find $y(0.8)$ by Adams-Bashforth formula. Find $y(0.2), y(0.4), y(0.6)$ by fourth order Runge-Kutta-method.

32. Given

$$\frac{dy}{dx} = 1 + y^2$$

where $y = 0$ when $x = 0$. Find $y(0.8)$ and $y(1.0)$ by Milne's formula.

33. The differential equation $y' = x^2 + y^2 - 2$ satisfies the following data:

x	y
-0.1	1.0900
0	1.0000
0.1	0.8900
0.2	0.7605

Use Milne's method to obtain the value of $y(0.3)$.

34. Using Adams-Bashforth predictor-corrector formulae, evaluate $y(1.4)$ if y satisfies

$$\frac{dy}{dx} + \frac{y}{x} = \frac{1}{x^2}$$

and $y(1) = 1, y(1.1) = 0.996, y(1.2) = 0.986, y(1.3) = 0.972$.

35. Find $y(2)$ by Milne's method if $y(x)$ is the solution of

$$\frac{dy}{dx} = \frac{1}{2}(x + y)$$

assuming $y(0) = 2, y(0.5) = 2.636, y(1) = 3.595, y(1.5) = 4.968$.

36. Tabulate by Milne's method the numerical solution of

$$\frac{dy}{dx} = x + y$$

from $x = 0.2$ to $x = 0.3$ given that

x	y
0	1
0.05	1.0526
0.1	1.1104
0.15	1.1737

37. Solve the differential equation

$$\frac{dy}{dx} = x^2 + y^2 - 2$$

given $y(-0.1) = 1.09, y(0) = 1, y(0.1) = 0.89$. Find $y(0.2)$ by series expansion and find $y(0.3)$ by Milne's method.

38. Solve the initial value problem

$$\frac{dy}{dx} = 1 + xy^2, \quad y(0) = 1$$

for $x = 0.4, 0.5$ by using Milne's method when it is given that

x	0.1	0.2	0.3
y	1.105	1.223	1.355

39. Using the Adams method, solve the simultaneous differential equations

$$\begin{aligned} \frac{dy}{dx} &= xy + z \\ \frac{dz}{dx} &= y - z \end{aligned}$$

with $y(0) = 0,\ z(0) = 1$.

40. Use Milne's method to solve the simultaneous differential equations

$$\begin{aligned} \frac{dy}{dx} &= x + z \\ \frac{dz}{dx} &= -xy \end{aligned}$$

with $y(0) = 1,\ z(0) = 0$.

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