Department of Mathematical and Computational Sciences
National Institute of Technology Karnataka, Surathkal

P. Sam Johnson nitksam@gmail.com

Computational Linear Algebra - MA 703
Problem Sheet 6

---

1. State whether the following are true or false and give a reason or a 2×2 or 3×3 counter example. Here $|A|$ denotes the determinant of matrix A.

   (a) The determinant of $A + I = (\texttt{det}A) + 1$.

   (b) The determinant of $A^4 = |A|^4$.

   (c) The determinant of $4A = 4|A|$.

   (d) The determinant of $A = |A|$.

   (e) If A is not invertible, then AB is not invertible.

   (f) The determinant of $AB = |A||B|$.

   (g) AB and BA have the same determinant.

2. If the determinants of R and S are 5 and -3 respectively, find the determinant of T, where

$$R = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \; S = \begin{pmatrix} e & f \\ c & d \end{pmatrix} \text{ and } T = \begin{pmatrix} 3a + 2e & 2c - 9a - 6e \\ 3b + 2f & 2d - 9b - 6f \end{pmatrix}.$$

3. For any values of a, b and c, find the determinant of the matrix $\begin{pmatrix} 1 & 1 & 1 \\ a & b & c \\ b+c & c+a & a+b \end{pmatrix}$.

4. Find the determinant of the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & \alpha + \beta & \alpha + \gamma \\ 1 & \beta + \alpha & 0 & \beta + \gamma \\ 1 & \gamma + \alpha & \gamma + \beta & 0 \end{pmatrix}.$$

   Moreover, if α, β, γ are the roots of the equation $x^3 = 1$, find the determinant of A.