

**Instructor :** P. Sam Johnson

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**Problem Sheet 1**

1. Construct a  $3 \times 3$  nonzero matrix  $A$  such that the vector  $\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$  is a solution of  $Ax = 0$ .
2. Construct three different linear systems  $Ax = b$  whose solution set is  $x_1 = -2$ ,  $x_2 = 1$  and  $x_3 = 0$ .
3. Find all the values of  $a$  if the following systems

$$x + y = 1; \quad 2x + ay = 2$$

- (a) has only one solution,
  - (b) infinitely many solutions and
  - (c) no solution.
4. Choose  $h$  and  $k$  such that the system

$$x + hy = 2p; \quad 4x + 8y = k$$

- (a) has only one solution,
  - (b) infinitely many solutions and
  - (c) no solution.
5. Under what condition on  $y_1, y_2, y_3$  do the points  $(0, y_1), (1, y_2), (2, y_3)$  lie on a straight line. What is an appropriate generalization of the result?
  6. Use Gaussian elimination to find a polynomial which passes through the following points  $(0,0), (1,4), (-1,0)$  and  $(-2,10)$ .
  7. If  $(a, b)$  is a multiple of  $(c, d)$  with  $abcd \neq 0$ , show that  $(a, c)$  is a multiple of  $(b, d)$ .

8. Find  $PA = LU$  factorization of the matrix  $\begin{pmatrix} -5 & 3 & 4 \\ 0 & 0 & -9 \\ 15 & 1 & 2 \end{pmatrix}$ .

9. Factor the following *tridiagonal* matrices into  $LU$  and  $LDU$  :  $\begin{pmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{pmatrix}$  and  $\begin{pmatrix} a & a & 0 \\ a & a+b & b \\ 0 & a & b+c \end{pmatrix}$ .

10. If  $A = \begin{pmatrix} 1 & -2 \\ -2 & 5 \end{pmatrix}$  and  $AB = \begin{pmatrix} -1 & 2 & -1 \\ 6 & -9 & 3 \end{pmatrix}$ , determine the first and second columns of  $B$ .