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National Institute of Technology Karnataka, Surathkal
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MA939 Functional Analysis
Problem Sheet - 7

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Answer **ALL** questions.

1. Show that every normed space X is homeomorphic to an open ball $B(0, r)$ for some $r > 0$.
2. Let U and V be subsets of a normed space X. If U is compact, V is closed and $U \cap V = \phi$, then prove that there exists $r > 0$ such that $[U + B(0, r)] \cap V = \phi$, where $B(0, r) = \{x \in X : \|x\| < r\}$.
3. Prove that two norms on a normed space are equivalent iff every Cauchy sequence with respect to one of the norms is a Cauchy sequence with respect to the other norm.
4. If Y is a proper closed subspace of ℓ_p $(1 < p < \infty)$, then show that there is an $x \in S_X$ so that distance $(x, Y) \geq 1$.
5. The Riesz lemma is generally not true for $r = 1$. Give an example.
6. Give three different examples of a closed and bounded set in a normed space which is not compact.
7. State and prove Arzela-Ascoli theorem.
8. A subset S of a metric space (X, d) is totally bounded iff every sequence in S has a convergent subsequence.
9. If the ball $B(0, r)$ in a normed space X is totally bounded, then prove that X is finite dimensional. (Hint : Riesz lemma)
10. Prove that the Riesz lemma is true for $r = 1$ when M is a finite dimensional subspace of a normed space X.
11. Define locally compact in a metric space. Prove that a normed space is finite dimensional iff it is locally compact.
12. Let Y be a finite dimensional subspace of a normed space X. Then for each $x \in X$, it is given that there is an element y_0 of Y such that $d(x, Y) = \|x - y_0\|$. Is the existence of y_0 unique?