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**Department of Mathematical and Computational Sciences**
**National Institute of Technology Karnataka, Surathkal**
**Odd Semester, 2013 - 2014**
**MA939 Functional Analysis**
**Problem Sheet - 6**

Date : 06.08.2013 Last Date of Submission : 07.08.2013

Answer **ALL** questions.

1. Define three-space property. Prove that finite-dimensionality and completeness satisfy three-space prperty.
2. If $U$ is compact and $V$ is closed, then prove that $U+V$ is closed. What happens if $U$ is also closed?
3. Let $M = \{(x_n) \in c_0 : x_m = mx_{m-1}$ for each even $m\}$ and $M = \{(x_n) \in c_0 : x_m = 0$ for each odd $m\}$. Prove that $M+N$ is a proper dense subspace of $c_0$ (with the sup norm).
4. Prove that any finite dimensional subspace of a normed space is closed.
5. It is given that $C^1[a,b]$ is Banach with respect to $\|x\|_1 = \|x\|_\infty + \|x'\|_\infty$ for $x \in C^1[a,b]$. Prove that the norm defined by $\|x\|_1 = |x(a)| + \|x'\|_\infty$ for for $x \in C^1[a,b]$ is a complete one.
6. Let $(X, \|.\|)$ be an infinite dimensional normed space and $\tau$ its topology. Show that there are two norms $\|.\|_1$ and $\|.\|_2$ on $X$ such that if $\tau_1$ and $\tau_2$ are the associated topologies for $\|.\|_1$ and $\|.\|_2$, then $\tau_2 \subseteq \tau \subseteq \tau_1$, $\tau_2 \neq \tau$, $\tau_1 \neq \tau$. That is, $\tau_1$ is strictly stronger than $\tau$, and $\tau_2$ is strictly weaker than $\tau$.
7. Let $1 \leq r < p \leq \infty$. Verify whether $\|.\|_p$ and $\|.\|_r$ on $c_{00}$ are equivalent or not.
8. Prove or disprove : In a finite dimensional normed space, all norms are equivalent.
9. Say true or false : $(C[a,b], \|.\|_p)$ is complete for $1 \leq p < \infty$.
10. Let $X$ be a metric space and $A$ be a subset of $X$. Prove or disprove the following :
    (a) If $A$ is compact, then $A$ is closed.
    (b) If $A$ is compact, then $A$ is bounded.
    (c) If $A$ is closed, then $A$ is compact.
    (d) If $A$ is bounded, then $A$ is compact.